Problem 3.22

Show that the energy-time uncertainty principle reduces to the "your name" uncertainty principle (Problem 3.15), when the observable in question is x.

Solution

Begin with the energy-time uncertainty principle.

$$\Delta t \, \Delta E \geq \frac{\hbar}{2}$$

By definition,

$$\Delta E = \sigma_H \quad \text{and} \quad \Delta t = \frac{\sigma_Q}{\left| \frac{d\langle Q \rangle}{dt} \right|}$$

for an observable Q. If $Q = x$, then the energy-time uncertainty principle becomes

$$\frac{\sigma_x}{\left| \frac{d\langle x \rangle}{dt} \right|} \sigma_H \geq \frac{\hbar}{2}$$

$$\sigma_x \sigma_H \geq \frac{\hbar}{2} \left| \frac{d\langle x \rangle}{dt} \right|$$

$$\sigma_x \sigma_H \geq \frac{\hbar}{2m} \left| m \frac{d\langle x \rangle}{dt} \right|.$$

Therefore, since $\langle p \rangle = m \, d\langle x \rangle / dt$,

$$\sigma_x \sigma_H \geq \frac{\hbar}{2m} |\langle p \rangle|.$$